



FEDERAL
RESERVE
BANK *of*
ST. LOUIS

A TWO-TRANCHE VIEW OF NATIONAL DEBT

**David Andolfatto, James Bullard, Riccardo
DiCecio and Guillaume Vandenbroucke**

Federal Reserve Bank of St. Louis

Peterson Institute for International Economics
Oct. 14, 2022
Washington, D.C.

Any opinions expressed here are our own and do not necessarily reflect those of the FOMC.

Introduction

RETHINKING NATIONAL DEBT

- Observed debt-to-GDP ratios in the U.S., Japan, and some large European economies appear to be unsustainably high according to conventional wisdom as embodied in the Maastricht Treaty.
- Nevertheless, the demand for this debt remains robust.
- We explore a possible resolution to this puzzle in a stylized model in which the equilibrium level of the debt-to-GDP ratio is considerably higher than what is recommended in the Maastricht Treaty.

WHAT WE DO

- We study a stylized DSGE “large” life-cycle model with interest-bearing government debt held as part of the competitive equilibrium.
- The fiscal authority:
 - Rolls over “first tranche” debt each period, which is unbacked by future taxes. “Interest-bearing money.”
 - Issues “second tranche” debt each period, which is paid off in the following period with labor income taxes. “Barro-Ricardian debt.”
 - The two tranches together, along with capital, provide the necessary level of assets to support the equilibrium.
- Enough realistic features to provide an interesting calibrated benchmark, including substantial inequality in income and financial wealth.

MAIN FINDING

- Main result: The federal debt-to-GDP ratio in the benchmark calibrated equilibrium of this economy is 123%, equal to the current U.S. gross federal debt-to-GDP ratio.
- Intuition: Our model provides a demand for nominally denominated government debt as a type of “interest-bearing money,” and the amount needed to support the competitive equilibrium turns out to be large.
- The results in this paper are best thought of as potentially applying to large stable economies with high fiscal and monetary policy credibility.
- Whether these results would apply to countries within the EMU is an open question.

REMARKS

- Complete characterization of optimal monetary/fiscal policy.
 - The fiscal authority issues one-period nominal debt at the expected rate of nominal output growth, and collects taxes to pay off a portion of previously issued debt.
 - The monetary authority observes shocks and then sets the price level to ensure that the real rate of return on this debt and other assets is equal to the stochastic rate of real output growth.
- The optimal fiscal-monetary mix works equally well whether interest rates are high or low.
- Monetary policy provides a form of insurance to investors by making the real return on government debt and other assets appropriately state-contingent in an economy with non-state-contingent nominal contracting.

MORE REMARKS

- In the equilibrium we describe, federal government debt coexists with capital and with privately-issued debt, which can be interpreted as mortgage-backed securities.
- We focus on one-period debt, but the model can price many-period debt.
- We abstract from international considerations, but this is an important area for future research.

BOTTOM LINE EXAMPLE

- The U.S. gross federal debt-to-GDP ratio is about 123% (Q1 2022).
- Standard macroeconomics would consider this entirely as Barro-Ricardian debt that must be backed by future taxation. Also, levels in excess of 60% (according to the Maastricht Treaty) are often considered excessive and suggest a future crisis.
- This paper calibrates a first tranche of 118%, which is “interest-bearing money” and not backed by future taxation.
- The second tranche of 5% is Barro-Ricardian debt that is backed by future taxes—a level that would normally be considered easily sustainable.
- The paper provides wide scope for interpreting how large the two tranches may be—the case with a first tranche of 118% presented here provides one illustration that the first tranche may be quite large.

Environment

MACROECONOMY WITH HETEROGENEOUS AGENTS

- At each date t , a new continuum of households enters the economy, makes economic decisions over $T + 1 = 241$ dates, then exits the economy.
- This corresponds to an agent entering the economy as a decision-maker at age 20 and exiting as a decision-maker at age 80, inclusive of end points, and making economic decisions at a quarterly frequency.
- Results are perfectly general for the choice of T , with higher values corresponding to decision-making at more frequent intervals.
- This class of models has a “paper-and-pencil” equilibrium solution, and so it provides a simple benchmark model for heterogeneous-agent macroeconomies with aggregate shocks.

RISK FACED BY HOUSEHOLDS

- There is both aggregate risk and idiosyncratic risk.
- All idiosyncratic risk is borne as the agent enters the economy.
- Macroeconomic policymakers provide a form of insurance against the aggregate risk.
- A welfare theorem describes the sense in which the equilibrium studied here represents a first-best allocation of resources.

ASSETS AND NOMINAL CONTRACTING

- There are three nominally denominated assets: privately issued debt, publicly issued debt, and capital.
- We think of these as representing U.S. data counterparts: (1) mortgage-backed securities, (2) federally issued debt and (3) physical capital, respectively.
- In the U.S. data, MBS net out, but federally issued debt and physical capital are in positive net supply and we target a value of assets-to-GDP equal to $1.23 + 3.32 = 4.55$.
- The credit market friction is non-state contingent nominal contracting (NSCNC): All debt contracts are stated in nominal terms, with a stated nominal interest rate, and repayment is not state-contingent.
- The role of monetary policy is to adjust the price level each period in order to convert these nominal, non-state contingent contracts into real, state-contingent contracts.

HOUSEHOLD TYPES

- Household types: “life cycle” and “hand-to-mouth.”
- The life-cycle households are assigned a hump-shaped productivity profile at the beginning of their life cycle. Accordingly, they need to use credit markets (hold assets) to smooth life-cycle consumption.
- The hand-to-mouth households are assigned a perfectly flat productivity profile as they enter the economy. Accordingly, they never need to use credit markets and instead consume their labor income each period.
- The economy with all life-cycle households wants to hold assets worth 5.71 times the size of GDP, higher than what we see in the data.
- The economy with all HTM households would be “Spartan,” and would hold no assets at all.
- We will adjust the fraction of HTM households in order to match the assets-to-GDP ratio in the U.S. data.

PREFERENCES

- Each household $i \in (0, 1)$ entering the economy at date t has preferences (both types)

$$U_{t,i} = \sum_{s=0}^T [\eta \ln \tilde{c}_{t,i}(t+s) + (1-\eta) \ln \ell_{t,i}(t+s)].$$

- We define $\tilde{c}_{t,i}(t+s) = D(t+s) c_{t,i}(t+s)$, where $D(t+s)$ is the state of aggregate demand at date $t+s$. The state of demand evolves as

$$D_t = \delta(t-1, t) D_{t-1},$$

where $\delta(t-1, t)$ is the gross growth rate of demand, which follows an appropriate stochastic process that keeps $D(t) > 0 \forall t$.

- Following Bai, Ríos-Rull and Storesletten ([unpublished, 2019](#)), we allow the state of aggregate demand to influence productive activity in the economy.

LIFE-CYCLE PRODUCTIVITY PROFILES

- Life-cycle households entering the economy draw a scaling factor x from a lognormal distribution and receive a life-cycle productivity profile that is a scaled version of the baseline profile, e_s :

$$e_{s,i} = x \cdot e_s,$$

where $e_s = 1 + p_1 \exp \left[- \left(\frac{s-p_2}{p_3} \right)^4 \right]$, and where p_1, p_2 and p_3 are chosen to match calibration targets given below.

- Huggett, Ventura and Yaron ([AER, 2011](#)) argue that differences in initial conditions are more important than differences in shocks.
- We also include a measure of hand-to-mouth agents in each cohort. They have a perfectly flat endowment profile, scaled up or down by a similar scaling factor.
- We think of all endowments as containing a labor tax factor $(1 - \tau)$.

TECHNOLOGY

- Aggregate real output $Y(t)$ is given by

$$Y(t) = [D(t) Q(t) N(t)]^{1-\alpha} K(t)^\alpha [L(t)]^{1-\alpha}, \quad (1)$$

where $K(t)$ is the real value of the physical capital stock, $L(t)$ is the aggregate effective human capital supply (hours \times productivity of various households), $Q(t)$ is a productivity index, $N(t)$ indexes the size of the labor force, and $D(t)$ is the state of aggregate demand.

- Q , N and D grow at stochastic gross rates λ , ν and δ respectively.
- These assumptions mean that real output grows at the stochastic rate $\lambda\nu\delta$ each period.
- The aggregate demand assumption is a simple version of Bai, Ríos-Rull and Storesletten ([unpublished, 2019](#)).
- The labor force growth assumption affects all cohorts proportionately and can be interpreted as “immigration.”

NOMINAL CONTRACTING AND TIMING PROTOCOL

- Under the assumptions outlined, the contract nominal interest rate is given by

$$R^n(t, t+1)^{-1} = E_t \left[\frac{\tilde{c}_{t,i}(t)}{\tilde{c}_{t,i}(t+1)} \frac{P(t)}{P(t+1)} \right]. \quad (2)$$

- The timing protocol is: (1) Nature assigns new entrant productivity profiles and also draws aggregate shocks; (2) The fiscal authority issues nominal debt; (3) The monetary authority sets the price level; (4) Households choose date t consumption, hours worked and net asset holding.
- Households will be able to make date t decisions without reference to future uncertainty, as the monetary policymaker is providing a type of perfect insurance.

THE FISCAL AUTHORITY

- The fiscal authority taxes all labor income in the economy at a fixed rate τ , and uses all of this revenue to pay off last period's Barro-Ricardian (BR) debt

$$B^{BR}(t-1) = P(t) \tau w(t) L(t). \quad (3)$$

- The fully credible nominal debt issuance process is given by $B(t) \equiv B^{IBM}(t) + B^{BR}(t)$ with

$$B(t) = R^n(t-1, t) B(t-1), \quad (4)$$

where $B(t)$ is the total nominal level of debt and $B(0) > 0$, and B^{IBM} refers to "interest-bearing money."

- The fiscal authority is issuing enough new debt of both types to maintain the level of assets in the economy at the appropriate level.

THE MONETARY AUTHORITY

- The monetary authority controls the price level directly and implements a price level path criterion

$$P(t) = \frac{R^n(t-1, t)}{\delta(t-1, t) \lambda(t-1, t) \nu(t-1, t)} P(t-1). \quad (5)$$

- This criterion implements countercyclical price level movements relative to the expectation embodied in the contract rate $R^n(t-1, t)$. See Koenig (*IJCB*, 2013) and Sheedy (*BPEA*, 2014).
- See our paper (p. 14) for a discussion of how this criterion relates to a similar New Keynesian “targeting criterion” developed by Giannoni and Woodford (2004, pp. 101-2).
- This monetary policy achieves the “Wicksellian natural real rate of interest” for this economy.

COMPETITIVE EQUILIBRIUM AND SOCIAL WELFARE

- Solution: Guess and verify that there is a competitive equilibrium in which the real rate of interest is always equal to the stochastic rate of real output growth.
- A social planner would conclude that the allocation of resources is a social optimum provided (i) the planner places equal weight on all households for all time, (ii) the planner discounts backward and forward in time at the stochastic real rate of interest, (iii) the planner cannot alter the distribution of productivity profiles within the cohort, which are decided by nature at the beginning of the life cycle, (iv) the planner cannot alter the tax rate τ .

Mapping to Data

MAPPING TO THE DATA

- Adjust cohort size based on data from the U.S. Census Bureau.
- Set the baseline hump-shaped life-cycle productivity profile such that households endogenously choose to work the hours worked by age in the U.S. data.
- Choose η to match average time devoted to market work across the economy.
- Set the fraction of HTM households (who do not hold assets) such that the aggregate level of assets to output, $A / (4Y)$, matches the U.S. data (4.55), with net assets defined as capital, $K / (4Y) = 3.32$, plus government issued debt, $B / (4Y) = 1.23$.
- Choose the within-cohort standard deviations of productivity for life-cycle and hand-to-mouth households, σ_{lc} and σ_{htm} , respectively, to approach the pre-taxes-and-transfers Gini coefficients for income and financial wealth in the U.S. data.

BASELINE LIFE-CYCLE PRODUCTIVITY

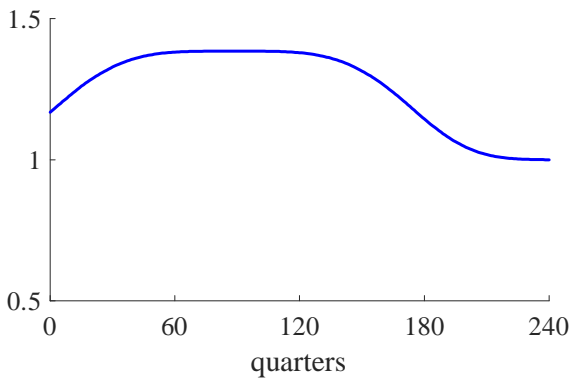


FIGURE: Baseline endowment profile of life-cycle agents.

THE MASS OF LIFE-CYCLE PRODUCTIVITY

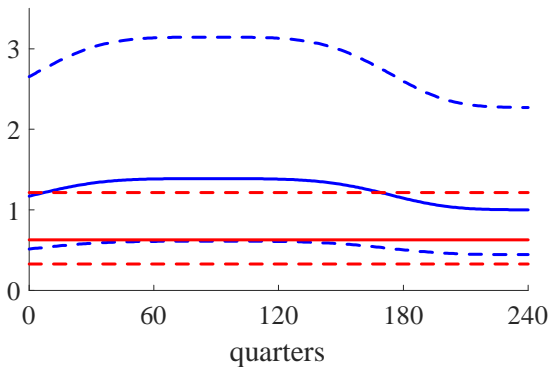


FIGURE: The mass of endowment profiles: life-cycle agents (blue) and hand-to-mouth agents for $h = 0.5$ (red). The dashed lines denote the 25th and the 75th percentile of the endowment distributions.

HOURS WORKED BY AGE

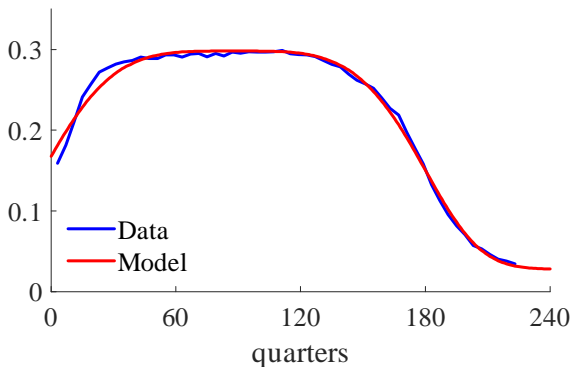


FIGURE: Hours worked by age: U.S. data (blue) and calibrated model (red).

POPULATION WEIGHTS

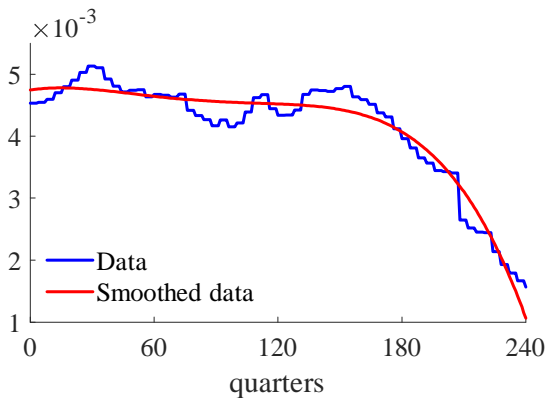


FIGURE: Population weights: U.S. data (blue) and 4th degree polynomial smoothed (red).

ASSETS AND GINI COEFFICIENTS

	Model			U.S. data
h	0.25	0.50	0.75	—
σ_{lc}	1.23	1.24	1.25	—
σ_{htm}	1.41	1.03	0.67	—
$A/(4Y)$	4.55	4.55	4.55	4.55
G_W	0.74	0.74	0.75	0.78
G_Y	0.66	0.66	0.66	0.63
G_C	0.62 [†]	0.62 [†]	0.62 [†]	0.32 [‡]

TABLE: Assets-to-output ratios and Gini coefficients: model vs. U.S. data.

[†] Pre-taxes-and-transfers consumption Gini.

[‡] Post-taxes-and-transfers consumption Gini.

THE CONSUMPTION GINI

- The *pre*-taxes-and-transfers consumption Gini in the model equilibrium is $G_c = 0.62$.
- In the U.S. data, the *post*-taxes-and-transfers consumption Gini is 0.32, about half as large.
- The model is saying that the net effect of taxes and transfers in the U.S. data is enough to reduce consumption inequality by half.
- Some evidence: Using German data, Haan, Kemptner, and Prowse ([working paper, 2018](#)) use a life-cycle model to estimate that the tax-and-transfer system is sufficient to offset 54% of the inequality in lifetime earnings.

NOMINAL RETURNS

- The model claims equalized nominal returns for three assets under optimal monetary policy: capital, MBS and Treasuries.
- These assets are not further differentiated inside the model.
- To compare to the data, we need an asset representing a return to capital in a format with risk characteristics similar to MBS and Treasuries.
- One candidate is a corporate bond index.
- The model equilibrium states that the nominal return on such an index should be equal to nominal consumption growth in periods of relative stability with optimal monetary policy.

NOMINAL RETURNS

	1997	2006	ANR	2010	2019	ANR
Corporate bond index	100	189	7.3%	100	165	5.7%
PCE	100	170	6.1%	100	145	4.2%
Services PCE	100	174	6.3%	100	148	4.5%

TABLE: Total annual nominal returns (ANR) on an index of corporate bonds vs. nominal consumption growth.

- The two time periods were relatively stable, with established monetary policy credibility.
- The differences in the bond index vs. the consumption index suggest some deviation from optimal monetary policy during these periods, according to the model.

Conclusions

HOW LARGE IS THE FIRST TRANCHE?

- This paper argues that some fraction of observed federal debt for large economies with substantial macroeconomic policy credibility may be of the first-tranche, “interest-bearing money” type. This fraction is not backed by future taxes.
- It does not cost anything, nor does it produce revenue.
- It can be rolled over in perpetuity at market interest rates; this process is going on in the background much as the replacement of worn currency is going on in the background.
- A second tranche of Barro-Ricardian debt could exist on top of the first tranche, and the marginal dollar of additional debt would then have to be backed by future primary surpluses.
- This would help explain why taking on additional debt is often politically contentious, while at the same time seemingly high raw federal debt-to-output ratios remain sustainable for some countries.